Scalable and Nonlinear Forecasting with Quantum ARMA Models

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Abstract
Time series forecasting is very important in planning, climate modelling, and industrial making-decision. It helps make plans and decisions. The Autoregressive Integrated Moving Average model (ARIMA) is a statistical method that is still widely used. It has some problems. It assumes that data changes in a line, which is not always true. This thing needs the data to be more stable which is not easy to do. Working with a lot of data can make it really slow. This study looks at a method called Quantum Autoregressive Moving Average (QARMA). QARMA uses quantum encoding, the Quantum Fourier Transform, and the Harrow–Hassidim–Lloyd algorithm to make forecasts. We used two datasets to test QARMA and compare it to ARIMA. One dataset had information on unemployment rates. The other had weather observations. The results show that QARMA works better than ARIMA with data that changes a lot. QARMA reduces errors in forecasts. Is still fast. The tests also confirmed that QARMA is reliable and works well. The findings suggest that using quantum computing for forecasting can make it better. QARMA can handle data and large amounts of it. This makes QARMA a promising new direction for analytics.
Keywords: *Time Series Forecasting, ARIMA, Quantum Computing, Quantum Machine Learning, QARMA.*

1 Introduction

1.1 Importance of Time Series Forecasting

Forecasting time series is essential for making decision in areas like economics, finance, climate science, and healthcare. Traditional statistical forecasting balances bias and variance for improved accuracy [5,11]. However, elevating data dimensions and nonlinear relationships pose difficulties for conventional linear models.

Recent developments in quantum computing have presented computational models that enable parallel state representation and high-dimensional feature mapping, providing possible benefits in forecasting applications [4,13].

1.2 ARIMA Model and Its Limitations

An ARIMA model is a statistical technique used to analysis and forecast future values in timeseries data based on its own past behaviour. It combines three components such as p – AutoRegressive to use past values, d- Integrated to make data stationary, and q – Moving Average to account for past forecast errors.

ARIMA (p, d, q) model is expressed as:

$$\phi(B)(1-B)^d Y_t = \theta(B)\epsilon_t$$

where:
$\phi(B)$ *is the autoregressive polynomial,*
$\theta(B)$ *is the moving average polynomial,*
d *is the differencing order, and*
ϵ_t *is white noise.*

ARIMA presumes covariance stationarity post-differencing and only represents linear relationships [4]. Its restrictions encompass:

- Difficulty in representing nonlinear dynamics
- Necessity for manual differencing
- Issues with scalability in extensive datasets
- Limitation to univariate analysis

Hybrid and nonlinear models, example ARIMA-ANN, ARIMA-SVM have been suggested to address these challenges [2,15], but they remain computationally intensive.

1.3 Quantum ARMA:

Quantum computing is really good, at handling lots of information because of entanglement and superposition. Quantum computing uses these ideas to make it work. A classical time series vector $x \in \mathbb{R}^n$ can be encoded as:

$$|x\rangle = \sum_{i=1}^{n} x_i \, |i\rangle$$

This amplitude encoding enables parallel representation of temporal features [14].

QARMA integrates:

- Quantum Fourier Transform (QFT) for frequency extraction [8]
- HHL algorithm for solving linear systems with potential exponential speedup [10]
- Variational Quantum Circuits (VQC) for nonlinear feature learning [7]

Under sparsity conditions, HHL reduces computational complexity from $O(n^3)$ to $O(\log n)$ in ideal scenarios.

1.4 Research Objectives

This study aims to compare classical ARIMA with Quantum ARMA to evaluate their effectiveness and practical applications. The key objectives are:

1. Compare the accuracy and efficiency of ARIMA and Quantum ARMA across different datasets.
2. Analysis how quantum principles like superposition and QFT improves forecasting.
3. Examine how Quantum ARMA can be used in social and climate modelling.
4. Provide insights on how businesses and researchers can integrate ARMA into their forecasting workflows.

By bridging the gap between traditional statistical methods and emerging Quantum computing techniques, this research aims to contribute to the next generation of Time Series forecasting.

1.5 Literature Review

Traditional principles to new quantum methodologies. Brockwell and Davis (1991), in their foundational book time series: Theory and Methods, laid out the essential theoretical framework for ARIMA models, explaining the management of stationary and not-stationary processes via autoregressive, moving average, and differencing elements, along with operational methods for model identification, estimation, diagnostics, and forecasting. This continues to serve as a standard for linear time series analysis. Subsequent research, including

Aslanargun et al. (2007), evaluated ARIMA against artificial neural networks and hybrid ARIMA-ANN models for predicting tourist arrivals, revealing that hybrid models frequently surpass pure linear ones by identifying nonlinear trends, thereby highlighting the constraints of isolated ARIMA in intricate datasets.

Lately, quantum-inspired techniques have started to investigate improvements for time series tasks. Daskin (2022), showed the viability of methods utilizing Quantum Fourier Transform (QFT) for preprocessing and forecasting, suggesting quantum counterparts to classical ARIMA elements that exploit superposition and entanglement for possibly more effective frequency-domain analysis and prediction. Foundational principles of quantum machine learning were presented by Schuld et al. (2015) in “An introduction to quantum machine learning”, covering data encoding, quantum optimization, and algorithmic opportunities. However, real-world application encounters major obstacles: Cerezo et al. (2021) pointed out issues such as noise, barren plateaus, and restrictions of NISQ devices, whereas Preskill (2018) characterized the NISQ era as a transitional stage allowing for exploratory yet noise-limited applications lacking complete fault tolerance.

Even with the rising fascination in quantum-enhanced forecasting- such as quantum ARIMA- like models and QFT- derived methods – direct empirical comparisons between classical ARIMA and quantum or quantum inspired options are still scarce. Many studies focus on theoretical feasibility or separate elements instead of thorough assessments of predictive accuracy, computational efficiency, scalability, or resilience across a variety of real-world datasets. The lack of thorough direct comparisons drives this research, aiming to fill the void via systematic testing and evaluation.

2 Methodology

A methodical approach for contrasting the ARIMA and QARMA models is described in this methodology. Along with discussing important issues, the section goes into each model's detailed procedures, tools, evaluation measures, and implementation tactics.

2.1 Dataset Descriptions

The first dataset, which has a record for 33 years and it's taken from Macrotrends, shows India's unemployment rates from 1991 to 2024. It forecasts future rates and aids in the examination of employment trends. The second dataset, which comes from data.gov (Utah FORGE), and it has 4,458 meteorological data recordings from the period of March 2024. It comprises variables which are crucial for researching the extraction of geothermal energy, such as temperature, wind direction, wind speed, humidity, and barometric pressure. To assess the prediction performance of the ARIMA and QARMA models on the provided dataset, they were developed in Python.

2.2 Experimental Protocol

Using an 80/20 train-test split, out-of-sample prediction, standard error metrics (RMSE and MAPE), and statistical significance testing of the performance difference, this section describes the systematic evaluation protocol used to compare the forecasting performance of the classical ARIMA model against the suggested QARMA approach on the same time series datasets.

The overall methodology adopted in this study follows a systematic workflow for time series forecasting using both classical and quantum-inspired models. **Fig.1** illustrates the complete process flow for ARIMA and QARMA model development, evaluation, and comparison.

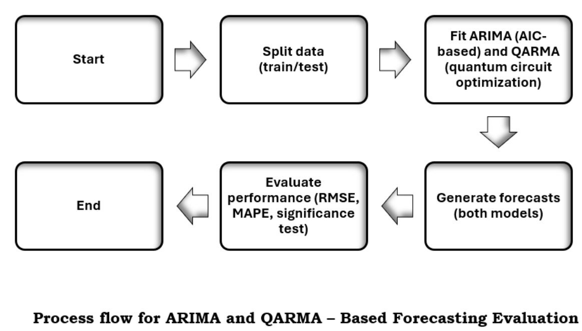

Fig. 1. Process flow chart

A simplified procedure for contrasting the quantum QARMA and classical ARIMA models in time series forecasting is shown in the flowchart. First, the dataset is divided into training and testing sets. Next, ARIMA is fitted using AIC-based parameter selection, and QARMA is fitted using quantum circuit optimization. After both models produce forecasts on the test set, statistical significance tests, RMSE, and MAPE are used to assess how well they performed. The process ends with a clear comparison of the two methods' forecasting resilience and accuracy.

2.1 ARIMA Model

The traditional autoregressive integrated moving average modelling or ARIMA modelling is done in a way. First, we check if the time series dataset follows stationarity. We do this with the Augmented Dickey-Fuller test. If the series is not stationarity, we make it stationary by detrending or by changing the data. We keep doing this until we get a series. This helps us find out how times we need to change the data, which is called the integration order d. When the series is stable, we try to figure out the orders of the autoregressive and moving average parts. We do this by looking at graphs of the autocorrelation function and partial autocorrelation function. The partial autocorrelation function graph usually stops after a point for an autoregressive process and the autocorrelation function graph stops after a certain point for a moving average process.

Then we use a method to estimate the parameters of the ARIMA model. Usually, we use the likelihood estimation method. The model parameters are estimated by applying the maximum likelihood method to fit the ARIMA model to the observed data. Subsequently, diagnostic checking is performed by analysing the residuals. Specifically, the Ljung-Box Q-test is employed to confirm that the residuals exhibit randomness and are free of serial correlation. If the residuals are not random, we go back. Adjust the parameters or change the data again. We keep doing this until we get a fit.

Finally, we use the ARIMA model to make forecasts for the test dataset. We also calculate the prediction intervals. The ARIMA model is used to generate these forecasts and intervals. The traditional ARIMA modelling is a tool for making predictions, about the future. The ARIMA model is used to forecast the values of the time series.

2.2 Quantum ARMA Modelling

By employing quantum superposition, entanglement, and efficient linear algebra calculations for time series forecasting, Quantum ARMA (QARMA) modelling seeks to extend conventional autoregressive moving average concepts to the quantum world. In order to represent the entire dataset (or a subset of recent observations) concisely in a superposition state, the procedure begins with amplitude encoding using quantum embedding techniques [14], in which classical time series values are allocated to the probability amplitudes of a quantum register. After that, this encoded state undergoes a quantum Fourier transform (QFT) [8], which performs a discrete Fourier transform in the quantum setting, moves the data from the time domain to the frequency domain, and makes it easier to retrieve periodic patterns and correlations that support autoregressive and moving average frameworks. The basic predictive relationships of the ARMA model, which are typically determined by solving a linear system for coefficients or future values, are then addressed by the Harrow-Hassidim-Lloyd (HHL) algorithm [10], which solves systems of linear equations exponentially faster than conventional methods (under certain conditions) by converting the matrix into a quantum operator, estimating eigenvalues using quantum phase estimation, and creating a quantum state whose amplitudes represent the solution vector.

Variational parameter optimization is carried out within a hybrid quantum-classical loop in order to adapt this framework for noisy intermediate-scale quantum hardware and improve the model parameters that describe the functions of the quantum circuit. This involves gradually adjusting differentiable circuit parameters using gradient-based techniques (such as the parameter-shift rule or finite-difference approximations) in order to lower a suitable loss function, such as prediction error on training data. Following optimization, the output quantum state is prepared and measured several times to estimate the expectation values of relevant observables. This process is known as quantum expectation measurement, and it is used to derive classical forecast values from the quantum superposition. The quantum ARMA process is ultimately completed by forecast generation, which yields point forecasts (and optionally uncertainty estimates) for the test set or future time periods. The technique is susceptible to circuit depth, gate errors, and measurement shot noise that are present in current quantum systems, despite being theoretically well-suited for handling complex temporal dependencies with reduced computational scaling under ideal circumstances.

2.3 Comparative Evaluation Metrics

Evaluation metrics are essential for comparing the forecasting accuracy and computational efficiency of ARIMA and QARMA models. Mean Squared Error (MSE) measures the how actual and predicted values are, with larger errors weighted more significantly [3]:
MSE is calculated as this:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y_1})^2 \qquad (1)$$

RMSE is helps to provide a direct comparison between the error and the original data, ensuring the results remain on a comparable scale.

$$\text{RMSE}=\sqrt{MSE} \qquad (2)$$

By expressing errors as a percentage of actual observations, MAPE shows errors as a percentage of values. This makes it easy to compare how well predictions do across datasets. By using MAPE you get a sense of performance that's not tied to the size of the data. It helps in comparing datasets of sizes.

$$\text{MAPE}=\frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i-\hat{y}_i}{y_i}\right| \times 100 \qquad (3)$$

In addition to accuracy, computational efficiency is another key factor. ARIMA models depend on repeated parameter estimation, which makes them costly in terms of computation when it comes to large datasets. In contrast, QARMA adopting the Quantum Fourier Transform (QFT), and the Harrow-Hassidim-Lloyd (HHL) algorithm, allowing for significant improvements in solving linear equations and handling high-dimensional data. This quantum-powered efficiency positions QARMA as a promising framework for scalable and high-performance time series forecasting.

2.4 Challenges and Opportunities

Quantum computing offers transformative possibilities but continues to contend with scalability issues. A primary obstacle is quantum noise, where even minor environmental disturbances compromise the integrity of quantum bits (qubits) and lead to processing errors. Another hurdle is scalability – right now, we don't have enough qubits to handle really complex problems, which limits what quantum models can do in real-world scenarios. Consequently, despite the transformative potential of quantum systems, a significant innovation gap remains before they reach a stage of commercial viability [7].

Conversely, ongoing advancements in error mitigation are bolstering computational reliability, while hybrid quantum-classical architectures offer a viable near-term framework for overcoming current hardware limitations These models allow us to tap into quantum power while still using traditional methods where quantum systems fall short. As these technologies improve, we're moving closer to making quantum computing a real game-changer in fields like time series forecasting and beyond.

QARMA shows great promise in improving time series forecasting by overcoming ARIMA's limitations with quantum computing. While challenges like noise and scalability still exist, ongoing advancements in hybrid models and error correction bring us closer to real-world applications.

3 Results and Discussion

3.1 Comparing Forecasting Accuracy and Efficiency

The Performance of the proposed Quantum-enhanced is evaluated against the ARIMA model with two different datasets and their values are displayed in **Table 1**. It presents the comparative results in terms of Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE) and training time.

Table 1. Content details

Dataset	Model	RMSE	MAPE (%)	Training Time(s)
Unemployment Rate	ARIMA	0.0121	16.98	0.054
	QARMA	0.0009	0.0603	0.159
Temperature (°F)	ARIMA	0.3043	0.67	0.627
	QARMA	0.8151	88.13	0.967

Dataset 1: Unemployment Rate

This dataset shows a trend making it good for simple forecasting models. QARMA works better. In terms, RMSE: QARMA has a low error rate of 0.0009 while ARIMA has 0.0121. This means QARMAs predictions are very close to values making forecast errors much smaller. MAPE: QARMA has an error of 0.0603% while ARIMA has 16.98%. This is a difference and QARMA is more accurate even when data is smooth. Training Time: ARIMA trains faster at 0.054 seconds compared to QARMA at 0.159 seconds. QARMA takes a bit time but it’s worth it for better forecasts. For economic indicators like unemployment rate QARMA works better than ARIMA in accuracy and is still reasonable, in computation time.

Dataset 2: Temperature (°F) (More Fluctuating & Seasonal Pattern)

This dataset contains stronger variability and environmental fluctuations. RMSE: ARIMA achieved a lower RMSE (0.3043) compared to QARMA (0.8151). This suggests that ARIMA provided more accurate predictions in terms of absolute error for this dataset. MAPE: ARIMA again performed better, with a MAPE of 0.67%, while QARMA showed a much higher 88.13%. The extremely high MAPE for QARMA indicates instability in percentage-based forecasting for this particular dataset. Training Time: QARMA required more time (0.967 seconds) compared to ARIMA (0.627 seconds). Here, the additional computational cost did not translate into improved forecasting performance. For temperature data with stronger variability and possible seasonal components, ARIMA demonstrates superior stability and forecasting accuracy. In this case, QARMA does not provide performance advantages and shows sensitivity to the data structure.

4 Conclusion

This study looked at how ARIMA and QARMA models can forecast things. It used two sets of data: one about the economy, which is the Unemployment Rate and one about the environment, which is Temperature in °F. The results show that QARMA is a lot better than ARIMA when the data is stable and structured. For the Unemployment Rate data QARMA did a better job of predicting what would happen next. This indicates that QARMA is highly predictive and adaptable to various contexts. Because QARMA was so much better at forecasting, it was worth the somewhat longer time it took to learn from the data.

The study also found out that QARMA is not always the model to use. ARIMA proved more effective for the temperature datasets, as it was better equipped to handle the frequent fluctuations and instability in the readings. ARIMA made mistakes and was more stable when forecasting the Temperature. QARMA made mistakes and took longer to compute even though it was not as good. Ultimately, model selection is contingent upon the underlying data structure; the chosen architecture must be compatible with the data's inherent properties. Perhaps in the future, we may be able to combine models like ARIMA with new quantum models like QARMA to create something that performs very well for a variety of data types.

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